M.Sc. 3rd Semester examination, 2019

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Partial Differential Equation and Generalized Functions)

Paper MTM – 301

FULL MARKS : 50 :: Time : 2 hours

1. Answer any four questions of the following: $2\times4 = 8$
   a. Define regular and singular distributions.
   b. What is the quasi-linear equation? Give an example.
   c. Give the definition of fairly good function with example.
   d. Eliminate the arbitrary function f and F from y = f(x- at) + F(x+at) .
   e. Find the PI of the PDE $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.
   f. What are the main difference between an ODE and PDE ?
   g. Find the adjoint of the differential operator L(u) = $u_{xx} + u_{tt} - u_t$.
   h. Define Dirac delta function.

2. Answer any four questions $4\times 4 = 16$

(a) Let u(x, y) be an integral surface of the equation
a(x, y)$u_x + b(x,y)u_y + u = 0$, where a(x, y) and b(x, y) are positive differentiable function in the entire plane. Define D = {(x, y): $|x| < 1 , |y| < 1$}

(i) Show that if u be positive on the boundary of D , then it is positive at every point in D.

(ii) Suppose that u attains a local minimum (maximum) at a point $(x_0, y_0) \in D$. Find $u(x_0, y_0)$.

(b) Show that if $\psi(x)$ is a fairly good function then $\psi(x)\delta(x) = \psi(0)\delta(x)$.

(c) Let $u \in C^2(D)$ be a function satisfying the mean value property. Show that **u** is harmonic in D.

(d) Establish the Laplace equation in polar coordinates.

(e) If $(\alpha_r D + \beta_r D' + \gamma_r)^2$ $(\alpha_r \neq 0)$ is a factor of F (D, D') and if the function ϕ_1 and ϕ_2 are arbitrary , then show that

$\exp\left(\frac{-x\,\gamma_r}{\alpha_r}\right) \sum_{i=1}^{2} x^{i-1} \phi_i(\beta_r x - \alpha_r y)$ is a solution of F (D, D') =0 .

(f) Prove that every nonnegative harmonic function in the disk of radius a satisfies

$$\frac{a-r}{a+r}\, u(0,0) \leq u(r,\theta) \leq \frac{a+r}{a-r}\, u(0,0).$$

(g) State the interior and exterior Dirichlet problem.

(h) Check the validity of the maximum principle for the harmonic function

$\frac{1-x^2-y^2}{1-2x+x^2+y^2}$ in the disk $\overline{D} = \{(x,y): x^2 + y^2 \leq 1\}$.

3. Answer any two questions 2×8

(a) (i) Show that the Green function for the Laplace equation is symmetric.

(ii)Establish the poisson's formula for the solution of a Dirichlet problem for the Laplace equation in a disk of radius a .

(b) Obtain the solution, valid when x, y > 0 , xy > 1 of the differential equation $\frac{\partial^2 z}{\partial x\, \partial y} = \frac{1}{x+y}$ such that z = 0, $\frac{\partial z}{\partial x} = \frac{2y}{x+y}$ on the hyperbola xy = 1.

(c) (i) Prove the following :

(a) $\delta(t) = \delta(-t)$.

(b) $\delta(at) = \frac{1}{a}\,\delta(t)$.

Symbols have their usual meaning.

(ii)Solve the following problem :

$$u_t = u_{xx} - u\,, 0 < x < 1\,, t > 0$$

$$u(0,t) = u_x(1,t) = 0\,, t \geq 0$$

$$u(x,0) = x(2-x), 0 \leq x \leq 1.$$

(d) (i) Show that the equations $xp - yq = x$ and $x^2p + q + xz$ are compatible.

(ii)Show that the equation $xpq + yq^2 = 1$ has complete integrals

(a) $(z + b)^2 = 4(ax + y)$

(b) $kx(z + h) = k^2y + x^2$ and deduce (b) from (a).

[Internal Assesment-10 Marks]

M.Sc. 3rd Semester examination, 2019

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Transformation and Integral Equations)

Paper MTM – 302

FULL MARKS : 50 :: Time : 2 hours

1. Answer any four questions of the following: $2\times4 = 8$
   a. Use the convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.
   b. Apply the convolution theorem to prove that B (m, n) = $\int_0^1 u^{m-1}(1-u)^{n-1}\ du = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, m>0 , n>0 .
   c. Define the term convolution on Fourier transform.
   d. Define the inversion formula for Fourier sine transform of the function f(x) . What happens if f(x) is continuous?
   e. State the Fredholm alternative concerning on integral equation.
   f. Define singular integral equation with an example .
   g. What is wavelet transformation?
   h. Write a short note on FBI fringerprint compression?

2. Answer any four questions

(a) Solve (tD^2+(1-2t)D -2) y = 0 ,y(0) =1, y^1(0) = 2 , where D$\equiv \frac{d}{dx}$.

(b) Show that if a function **f(x)** defined on $(-\infty, \infty)$ and its Fourier transform F(ζ) are both real, then **f(x)** is even. Also show that if f(x) is real and its Fourier transform **F(ζ)** is purely imaginary, then **f(x)** is odd.

(c) If the function f(t) has the period T > 0 then prove that

L**{f(t)}** = $\frac{1}{1-e^{-pT}}\int_0^T f(t)e^{-pt}dt$.

(d) Discuss the solution procedure for solving the homogeneous Fredholm integral equation with seperable kernel.

(e) Find the eigen value and corresponding eigen functions of the integral equation . $y(x) = \lambda \int_0^{2\pi} \sin x \cos t \, y(t)dt$.

(f) If the Fourier transform of f(x) is $\frac{\alpha}{1+\alpha^2}$, α being the transform parameter , then find f(x).

(g) Show that if ψ is wavelet and ϕ is bounded integrable function , then the convolution function $\psi * \phi$ is a wavelet.

(h) Prove that Haar wavelet is one of the most fundamental example of the general wavelet theory.

3. Answer any two questions 2×8

(a) Convert $y''(x) - 3y'(x) + 2y(x) = 4\sin x$ with initial condition y(0) =1, $y'(0) = 2$ into a volterra integral equation of the second kind . Conversely derive the original differential equation with initial conditions from the integral equation obtained.

(b) (i) Solve the integral equation $y(x) = f(x) + \lambda \int_{-1}^{1} (xt + x^2t^2)\, y(t)dt$.

(ii) Evaluate $L\{\int_0^t \frac{\sin u}{u}\, du\}$ by the help of initial value theorem.

(c) State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{ab(a+b)}$, a, b > 0.

(d) Using the Fourier sine transform, solve the PDE $\frac{\partial v}{\partial t} = k\,\frac{\partial^2 v}{\partial x^2}$ for x > 0, t > 0 , under the boundary conditions $v = v_0$ when x = 0 , t > 0 and the initial condition v = 0 , when t = 0 , x > 0 .

[Internal Assesment-10 Marks]

M.Sc. 3rd Semester Examination, 2019

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

Paper MTM – 303

FULL MARKS : 50 :: Time : 2 Hour

Unit-I (Dynamical Oceanology and Meteorology)

1. Answer any two questions $2\times 2=4$
   a) What is the difference between Synoptic Oceanography and dynamical Oceanography.
   b) Define salinity and sigma-t for sea water.
   c) What is mixing ratio.
   d) Define dry adiabatic lapse rate and write its value.

2. Answer any two questions $4\times 2=8$

   a) Find the relation between entropy and potential temperature.
   b) Prove that, the path of the particle describes an ellipse of a progressive wave in the surface of a cannel of finite depth.
   c) Derive the basic statical relationship of meteorology. Hence find out the pressure variation with altitude for temperature (T) is constant with height (z) and T decreases at a constant rate with increasing z.
   d) i) Describe the relation between Salinity Temperature and variation of density with graphical approach.
   ii) Define Secondary forces in oceanlogy. 3+1

3. Answer any one questions $8\times 1=8$

a) Derive hypsometric equation. Prove that the total energy of progressive wave is $\frac{1}{2}\rho g a^2 \lambda$ where a, λ is the wave amplitude and wave length respectively. 3+5

b) i) Write down the equation of motion in oceanology. How pressure term can be obtained form function of motion in oceanology? 1+3

ii) Derive the equation of continuity of volume in oceanology. Also write its form when field is incompressible. 3+1

Unit-II (Special Paper: Operations Research)

1. Answer any two question of the following: $2\times 2 = 4$
   a. Explain traffic intensity related to queueing model.
   b. What are the shortage and holding costs associated with inventory? Explain it.
   c. Explain what is meant by Kuhn-Tucker conditions?
   d. Explain : Demand and Lead time.

2. Answer any two of the following: $2\times 4 = 8$

(a) Find the optimum order quantity for a product for which the price breaks are as following :

Range of quantity	Unit Price
0 < Q < 100	Rs. 20
100 <= Q < 200	Rs. 18
200 <= Q	Rs. 16

The monthly demand for the product is 400 units. The shortage cost is 20% of the unit cost of the product. The cost of ordering is Rs. 25 per order.

(b) Derive the differential difference equation for (M/M/1: ∞/FCFS/∞) queueing system in transient stage.

(c) Using method of constrained variation, find the extrema point:

Min $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2$

Subject to $2x_1 + 4x_2 + 3x_3 - 9 = 0$

$4x_1 + 8x_2 + 5x_3 - 17 = 0$

(d) The demand for an item in a company is 1800 units per year. The company can produce the item at a rate 3000 per month. The cost of one set-up is Rs. 500 and the holding cost of one unit per month is Rs. 0.15. The shortage cost of one unit is Rs. 20 per month. Determine the optimum manufacturing quantity and shortage quantity. Also determine the manufacturing time and time between setup.

3. Answer any one question $1\times 8 = 8$

(a) Solve using Kuhn-Tucker condition

Max $Z = 5 + 8x_1 + 12x_2 - 4x_1^2 - 4x_2^2 - 4x_3^2$

Subject to $x_1 + x_{2,} <= 1$

$2x_1 + 3x_2 <= 6$

(b) A super market has two girls ringing up to sell at the counters. If the service time for each customer is exponential with mean 4 minutes and if the people arrive in a poisson at fashion at the counter at the rate 10 per hour, then

(i) What is the probability of having to wait for service?

(ii) What is the expected percentage of idle time for each girl ?

(iii) If a customer has to wait, what is the expected length of his waiting time ?

[Internal Assesment-10 Marks]

M.Sc. 3rd Semester examination, 2019

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Advance Optimization and Operations Research)

Paper MTM – 305

FULL MARKS : 50 :: Time : 2 hour

1. **Answer any four questions of the following:** $\mathbf{4\times 2 = 8}$

(a) Define integer programming problem? Give an example of it.

(b) Write the limitation of Fibonacci Method?

(c) Define quadratically convergent method and A-conjugate directions.

(d) Explain Different types of achievements in goal programming problem.

(e) Define unimodal maximization and minimization function.

(f) Using algebraic approach show that the expression $ax + \frac{b}{x} + c; a, b > 0$ $as\ minimum\ value$ $2\sqrt{ab} + c$ at $x = \sqrt{\frac{b}{a}}$.

(g) “Revised simplex method is better than the original simplex method “ ,why?

(h) What is post optimality analysis?

2. **Answer any four questions of the following:** $\mathbf{4\times 4 = 16}$

(a) Write the procedure of Fibonacci method to slove a unimodal optimization problem.

(b) Find the 1st Gomory ‘s constraints of the following integer programming problem

Maximize z= $3x_1 - 2x_2$

Subject to $12x_1 + 7x_2 \leq 28$ $x_1, x_2 \geq 0$ and are integers.

(c) Find the Conjugate directions for the Symmetric matrix $\begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}$

(d) The production manager facts the problem of job allocation among three of his teams. The processing rates of three teams are 5,6, and 8 units per hour respectively .The normal Working hours for each team are 8 hours per day. The Production manager has the following goals for the next day in order of priority :

(i) The manager wants to avoid any underachievement of production level, which is set at 180 units of production.

(ii) Any overtime operation of team 2 beyond 2hrs and team 3 beyond 3 hrs should be avoided.

(iii) Minimize the sum of overtime.

Formulate above goal programming problem.

(e) Using Newton's method

Minimize $f(x_1, x_2) = x_1 - x_2 + 2{x_1}^2 + 2x_1x_2 + {x_2}^2$ with (0,0) as starting point.

(f) When required an artificial constraint method to slove an LPP . explain it with an example.

(g) For the LPP

Maximize $z = 3x_1 + 5x_2$

Subject to $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 1 \qquad x_1, x_2 \geq 0$

The optimal table for the simplex method is found to be

c_B	y_B	X_B	x_1	x_2	x_3	x_4
0	x_1	$\frac{2}{3}$	$\frac{1}{3}$	0	1	$\frac{-1}{3}$
5	x_2	$\frac{1}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$
		$\frac{5}{3}$	$\frac{1}{3}$	0	0	$\frac{5}{3}$

Obtain the ranges of c_j for which the optimal solution remains optimal when changes one at a time .

(h) write the steps of Davidon – Fletcher –Powell method to slove a non linear optimization problem.

3. **Answer any two questions of the following:** $\mathbf{8 \times 2 = 16}$

(a) Solve the following IPP using Branch and bound method.

Max $z = 7x_1 + 9x_2$

Subject to $-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35 \qquad x_1, x_2 \geq 0$ and integers.

(b) Solve the following LPP by revised simplex method

Minimize $z = 2x_1 + x_2$

Subject to constraints

$3x_1 + x_2 \leq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$ and $x_1, x_2 \geq 0$

(c) Using cutting plane method ,slove

Maximize f = $7 - 2x_1 - 4x_2$

Subject to the constraints

$(x_1 - 4)^2 + 2(x_2 - 3)^2 - 12 \leq 0$

$x_1 + 2x_2 - 6 \leq 0$

$1 \leq x_1, x_2 \leq 6$ with the tolerance $\varepsilon = 0.03$

(d) Solve the following goal programming problem.

Minimize z= $d_1^- + d_2^- + d_3^-$

Subject to constraints

$20x_1 + 10x_2 \leq 60$

$10x_1 + 10x_2 \leq 40$

$40x_1 + 80x_2 + d_1^- - d_1^+ = 1000$

$x_1 + d_2^- - d_2^+ = 2$

$x_2 + d_3^- - d_3^+ = 2$ $x_1, x_2, d_i^- \, d_i^+ \geq 0$ $i = 1,2,3$

Internal Assessment : 10 Marks

M.Sc. 3rd Semester examination, 2019

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Special Paper: Operational Research Modelling –I)

Paper MTM – 306

FULL MARKS : 50 :: Time : 2 Hours

1. Answer any four questions of the following: $2\times4 = 8$
   a. What is critical path? What are the main features of it?
   b. What do you mean by time-cost trade off?
   c. What do you mean by supply chain management (SCM)? What is the main Objective of SCM?
   d. What is the Limitations of simulation?
   e. What is the basic idea of dynamic programming ?
   f. Explain the team's 'optimistic time' and 'most likely time' in PERT network.

   g. What is Bellman's principle of optimality?

   h. State Mortality theorem related to replacement management.

2. Answer any four questions 4×4

a. Derive the conditions that determine the optimal period of replacing an item whose maintenance cost is increase with time (discrete quantity) but money value is unchanged.

b. A project consists of eight activities with the following relevant information.

Activity	Time estimates(days)			Predecessor
	t_0	t_m	t_p	
A	**1**	**1**	**7**	**None**
B	**2**	**4**	**7**	**None**
C	**2**	**2**	**8**	**None**

D	**1**	**1**	**1**	**A**
E	**2**	**5**	**14**	**B**
F	**2**	**5**	**8**	**C**
G	**3**	**6**	**15**	**D, E**
H	**1**	**2**	**3**	**F, G**

(i) Draw the network and find the expected project completion time.

(ii) If the duration for activity F increases to 14 days what will be its effect on this expected project.

(c) A bakery keeps stock of a popular brand of cake. Previous experience shows the daily demand pattern for the item with associated probabilities as given below:

Daily demand	0	10	20	30	40	50
probability	0.01	0.20	0.15	0.50	0.12	0.02

Random number: 40,19,87,83,73,84,29,09,02,20: use the following sequence of random numbers to simulate the demand for next 10 days.

(d) Describe the kind of problems for which the Monte Carlo will be an appropriate method of solution.

(e) Write the steps to solve the following problem using dynamic programming technique Maximize $Z = f_1(x_1).f_2(x_2).\ldots f_n(x_n)$

Subject to $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b; a_j, x_j$, b$\geq 0$ $for\ j = 1,2, \ldots..n$

(f) The cost of a new machine is Rs.5000.00. The maintenance cost of nth year is given by $c(n) = 500(n-1); n = 1,2,3$ .... suppose that the discount rate per year is 0.05, after how many years will it be economical to replace the machine by a new one ?

(g) Find the sequence that minimizes the total elapsed time required to complete the following tasks

Tasks	A	B	C	D	E	F	G	H	I
Time of machine –I	2	5	4	9	6	8	7	5	4

Time of machine-II	6	8	7	4	3	9	3	8	11

(h) A newspaper-boy buys papers for rs. 1.70 each and sells them for rs. 2.00 each. He can not return unsold newpapers. Daily demand has the following distribution;

No. of customers	23	24	25	26	27	28	29	30	31	32
Probability	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05

If each day's demand is independent of the previous day's, how many papers he should order each day.

3. Answer any two questions 2×8

(a) (i) Formulate and solve a single period discrete stochastic inventory model for a single product with instantaneous discrete demand, zero lead time and no replenishment cost. The storage and shortage costs are independent of time. A firm is considering replacement of a machine, whose cost price is rs. 12200 and the scrap value Rs. 200. The running costs are found from experience to be as follows

(ii)

Year	1	2	3	4	5	6	7	8
Running cost Rs.	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced.

(b) Suppose in a system all items are new at beginning. Each item has a probability p of failing immediately before the end of the first month of life and probability $q(= 1 - p)$ of failing immediately before the end of the second month. If all items are as they fail. Show that the expected number of failure $f(x)$ at the end of month is given by $f(x) = \frac{N}{1+q}[1 - (-q)^{x+1}]$ where N be the initial items of the system.

If the cost per item of individual replacement policy is Rs. C_1 and the cost per item of group replacement policy is Rs. C_2. Find the condition under which group replacement policy at the end of the first month is most profitable over individual replacement.

(c) Solve the following LPP by dynamic programming approach

Maximize $Z=8x_1+7x_2$

Subject to the Constraints

$2x_1+x_2 <=8$ and $5x_1+2x_2 <=15$, $x_1, x_2 >=0$.

(d) A team of software developers at Microsoft is planned to rise to a strength of 50 persons and then to remains at that level. Consider the following data:

Year	1	2	3	4	5	6	7	8	9	10
Total % who have left upto the end of the year	5	30	50	60	70	75	80	85	90	100

On the basis of above information, determine

i) What is the recruitment per year necessary to maintain the strength?
ii) There are 8 senior post's for which the length of service is the main criterion, what is the average length of service after which a new entrant can expect his promotion to one of these post.

[Internal Assesment-10 Marks]